

# TAGMaC Abstracts

November 2019

## Plenary Speaker - Mette Olufsen

**Institution:** NCSU

**Title:** How mathematical techniques can be used to better understand cardiovascular dynamics in health and disease

**Abstract:** Dating back to the 1600s mathematics has been used to study cardiovascular dynamics enabling scientist to answer essential questions. In fact, todays knowledge that the cardiovascular system is circulating was first discovered via a mathematical model. I will discuss the role mathematical analysis has played in cardiovascular physiology and how to use mathematics to analyze data and study questions that cannot be answered from experiments alone. Several topics will be highlighted including how to merge imaging with dynamic data to understand what vessels should be opened with balloons for patients with pulmonary hypertension, how to understand what happens in patients with postural orthostatic hypertension, and how to use modeling to develop a new method for early detection of sepsis. Mathematical techniques used involve modeling (ODEs and PDEs), signal processing, sensitivity analysis and parameter estimation, and topological data analysis.

## Mini-Symposium 1: Analysis of PDE's

**Author:** Russel Arnold

**Institution:** UNC

**Title:** Hyperbolic Systems of Conservation Laws

**Abstract:** In this expository talk, Lax's construction for solutions of the Riemann problem for systems of hyperbolic conservation laws will be outlined after motivation is provided by considering the case of a single conservation law. For illustration, a concrete example will be computed from the equations of isentropic gas dynamics in a tube and time permitting, solutions with more general initial conditions will be discussed.

**Author:** Clair Kiers

**Institution:** UNC

**Title:** A Bifurcation Analysis of Standing Pulses and the Maslov Index

**Abstract:** The Maslov index is a powerful and insightful tool that can be used to determine the stability of solutions for PDEs. We demonstrate the robustness of a certain method of Maslov index calculation by applying it to standing pulse solutions of a three-component reaction-diffusion system. The Maslov index shows exactly why the stability of a wave changes at a bifurcation due to the appearance of a conjugate point.

**Co-Author:** Paul Cornwell

**Author:** Blake Keeler

**Institution:** UNC

**Title:** Spectral Asymptotics on Manifolds without Conjugate Points

**Abstract:** The eigenvalues and eigenfunctions of a manifold can be thought of as frequencies and modes of vibration. These eigenpairs describe essentially everything you might want to know about heat and wave propagation, as well as the probability densities of quantum particles. As a result, the entire field of spectral geometry has been developed for the purpose of studying these eigenpairs in a generic setting. In this talk, we will be primarily concerned with the "high frequency" behavior, which means we look at what happens as the eigenvalues become very large. We will give a general overview of the rich history of high-frequency spectral asymptotics, and we will discuss a new theorem which generalizes previous results in the case where our manifold has no conjugate points.

## Mini-Symposium 2: Algebraic and Complex Geometry

**Author:** Paul Kruse

**Institution:** UNC

**Title:** Stability Conditions on K3 surfaces and Some Wall Crossings

**Abstract:** The study and classification of vector bundles on manifolds, and more generally sheaves on complex varieties, has motivated the study of moduli spaces. To construct these moduli space, various notions of stability have been developed. In particular, slope stability and Gieseker stability have proven to be effective in many such cases. Recently, Bridgeland introduced a more generalized notion of stability that has furthered the study of moduli spaces. In this talk, I will introduce Bridgeland Stability Conditions on K3 surfaces, and present an application of their use in describing a particular moduli space of sheaves

**Author:** Georgy Scholten

**Institution:** NCSU

**Title:** Log Concave MLE

**Abstract:** Maximum likelihood estimators are commonly used in non-parametric statistics to find the best fitting probability distribution to a weighted sample set of points  $X$ . We impose that the logarithm of the optimal density function is concave and piecewise linear on the cells of a regular polyhedral subdivision of the configuration  $X$ . The focus of this talk will be on the geometry and combinatorics of the underlying optimization problem.

**Co-Authors:** Kaie Kubjas, Alex Grosdos, Alexander Heaton, Olga Kuznetsova, Stefana Sorea, G.S.

**Author:** Hunter Dinkins

**Institution:** UNC

**Title:** Quasimaps to a Nakajima Quiver Variety

**Abstract:** Nakajima quiver varieties form a family of smooth symplectic varieties with a rich structure and strong ties to representation theory and mathematical physics. Yet as objects arising from quivers, much of their geometry can be studied explicitly and combinatorially. In this talk, I will give an overview of the definition and basic properties of Nakajima quiver varieties and will define an enumerative invariant known as the Vertex function, which roughly speaking, gives an equivariant count of quasimaps to a quiver variety. Time permitting, I will also state a result providing a factorization formula for the Vertex function of type-A quiver varieties in terms of the well-known  $q$ -binomial series.

**Co-Author:** Andrey Smirnov

## Mini-Symposium 3: Mathematical Biology

**Author:** Ben Vadala-Roth

**Institution:** UNC

**Title:** Stabilization of the Immersed Boundary Method for Finitely Thin Structures

**Abstract:** The hybrid immersed boundary (IB) method is a model for incompressible fluid-structure interaction (FSI) that uses finite differences on a Cartesian grid to approximate the Eulerian equations, finite elements (FE) on an unstructured mesh to approximate the Lagrangian equations, and regularized delta functions to transfer data between the two frames. Unfortunately, the method encounters issues when the structure's physical thickness becomes very thin, especially in comparison to the background Eulerian grid spacing. We explore a possible strategy for ameliorating this issue.

**Co-Authors:** Simone Rossi, Boyce Griffith

**Author:** John Lagergren

**Institution:** NCSU

**Title:** Learning PDEs from Noisy Spatiotemporal Data

**Abstract:** We investigate methods for learning partial differential equation (PDE) models from spatiotemporal data under biologically realistic levels and forms of noise. Recent progress in learning PDEs from data have used sparse regression to select candidate terms from a denoised set of data, including approximated partial derivatives. We develop a novel methodology that uses artificial neural networks (ANNs) to denoise data and approximate partial derivatives. The methodology is tested on two PDE models for biological transport, i.e., the advection-diffusion and classical Fisher-KPP equations. We show that the ANN methodology outperforms previous denoising methods, including finite differences and polynomial regression splines, in the ability to accurately approximate partial derivatives and learn the correct PDE model.

**Co-Authors:** John T. Nardini, G. Michael Lavigne, Erica M. Rutter, Kevin B. Flores

**Author:** Andrew Ford

**Institution:** UNC

**Title:** Simulating Polymer-Like Substances in Biology

**Abstract:** As research advances in biological and medical fields, there is a growing need for simulations which can be studied. For certain long biological molecules, it is possible to generate simulations using modeling techniques originally developed for the study of molecular dynamics. This talk will cover the application of molecular dynamics techniques to the study of chromosome conformation dynamics and mucus modeling. It will cover the process of choosing and developing a project, the relevant quantities we will use the models to study, and some preliminary results.

## Mini-Symposium 4: Mathematical Modeling

**Author:** Gracie Conte

**Institution:** UNC

**Title:** Discretizing Schrodinger Type Operators with Spectral Accuracy on Quantum Graphs

**Abstract:** The study of quantum graphs is motivated both by direct applications of the graph models to physical phenomena and by use of graphs as a simpler setting in which to study complex phenomena of quantum mechanics. I will describe some basic tools in the spectral theory of Schrodinger type operators on metric graphs by studying some basic examples. A tool of especial importance that will be discussed in detail is spectral discretizations based on rectangular differentiation matrices.

**Author:** Kathrine Daftari

**Institution:** UNC

**Title:** Using Stochastic Models to Understand Single Particle Microrheology

**Abstract:** Stochastic processes provide a simple, yet robust model for a variety of biological mechanisms. One such model, Fractional Brownian Motion (FBM), provides a good approximation for passive particle motion in complex biological fluids at specific time scales. Fitting such a model to experimental datasets motivates estimation of key rheological parameters which can be used to classify disease progression in patients with cystic fibrosis. This talk will introduce the experimental setup and the subsequent data analysis techniques with their associated challenges and open questions.

**Co-Author:** Neall Caughman

**Author:** Dylan Bruney

**Institution:** UNC

**Title:** Faxen's First Law: The Challenging Exercise of Validating a 100 Year Old Theorem

**Abstract:** For this presentation, I will derive Faxen's First Law for a sphere; an equation that connects the force on a sphere's surface to the value of the incident flow and the Laplacian of incident flow at the sphere's center. It will be expository in that there is some gaps in logic I still need to fill. It is my intent that the audience will leave with the impression that simple looking Theorems can be deceptively long and hard to prove.

## Mini-Symposium 5: Algebraic Structures

**Author:** Thomas Tran

**Institution:** Duke

**Title:** Secondary Terms in Asymptotics for the Number of Zeros of Quadratic Form

**Abstract:** Let  $F$  be a non-degenerate quadratic form on an  $n$ -dimensional vector space  $V$  over the rational numbers. One is interested in counting the number of zeros of the quadratic form whose coordinates are restricted in a smoothed box of size  $B$ , roughly speaking. For example, Heath-Brown gave an asymptotic of the form:  $c_1 B^{n-2} + O_{J,\epsilon,\omega}(B^{(n-1)/2+\epsilon})$ , for any  $\epsilon > 0$  and  $\dim V \geq 5$ , where  $c_1 \in \mathbb{C}$  and  $\omega \in \mathcal{S}(V(\mathbb{R}))$  is a smooth function. More recently, Getz gave an asymptotic of the form:  $c_1 B^{n-2} + c_2 B^{n/2} + O_{J,\epsilon,\omega}(B^{n/2+\epsilon-1})$  when  $n$  is even, in which  $c_2 \in \mathbb{C}$  has a pleasant geometric interpretation. We consider the case when  $n$  is odd and give an analogous asymptotic of the form:  $c_1 B^{n-2} + c_2 B^{(n-1)/2} + O_{J,\epsilon,\omega}(B^{n/2+\epsilon-1})$ , where the geometric interpretation of the constant  $c_2$  of the asymptotic is strikingly different.

**Author:** Logan Tatham

**Institution:** UNC

**Title:** Representations of Link Invariants and their Link to Quantum Groups

**Abstract:** In this talk, we will introduce the notion of diagrammatic calculus and give several examples. We will then introduce the notion of a quantum group. Finally, we will tie the two ideas together and see their application to the construction of link invariants. As an example, we will display some new results in the case of type C Lie algebras.

**Co-Author:** David Rose

**Author:** Grant Barkley

**Institution:** NCSU

**Title:** Coxeter groups and the lattice of total orders

**Abstract:** A Coxeter group is a group generated by elements of order 2, called reflections, along with certain relations between the elements. The group of permutations on  $n$  elements (generated by swapping 1 and 2, 2 and 3, etc.) is an important example of a Coxeter group. Each permutation has a unique set of inversions; we may define a partial order, called weak order, on permutations by ordering these sets by containment. This order has many important properties, such as being a semidistributive lattice. We may define inversions and weak order similarly for arbitrary Coxeter groups, but when the group has infinite size, the weak order is no longer a lattice. It has been conjectured by Dyer and others that, by considering instead of inversion sets a generalization called biclosed sets, we re-obtain a lattice with similar properties to the weak order on permutations. We prove this conjecture for classical affine Coxeter groups by interpreting permutations as total orders.

**Co-Author:** David Speyer

**Author:** Stephen Lacina

**Institution:** NCSU

**Title:** Poset Topology of  $s$ -weak Order

**Abstract:** We introduce a generalization of weak order on permutations called  $s$ -weak order due to Ceballos and Pons. This is a lattice on certain labeled trees. We prove the order complexes of open intervals in  $s$ -weak order are either contractible or homotopy equivalent to a sphere of some dimension. We do this by giving  $s$ -weak order a certain edge labeling known as an SB-labeling the notion of which is due to Hersch and Mészáros. Additionally, we characterize the intervals which are homotopy spheres and the dimension of those spheres.

## Mini-Symposium 6: Statistics and Probability

**Author:** Steven Gilmore

**Institution:** NCSU

**Title:** A Model of Debt with Bankruptcy Risk and Currency Devaluation

**Abstract:** We consider a system of Hamilton-Jacobi equations, arising from a stochastic optimal debt management problem in an infinite time horizon with exponential discount, modeled as a non-cooperative interaction between a borrower and a pool of risk-neutral lenders. In this model, the borrower is a sovereign state that can decide how much to devalue its currency and which fraction of its income should be used to repay the debt. Moreover, the borrower has the possibility of going bankrupt at a random time and must declare bankruptcy if the debt reaches a threshold  $x^*$ . When bankruptcy occurs, the lenders only recover a fraction of their capital. To offset the possible loss of part of their investment, the lenders buy bonds at a discounted price which is not given a priori. This leads to a nonstandard optimal control problem. We establish an existence result of solutions to this system and in turn recover optimal feedback payment strategy  $u^*(x)$  and currency devaluation  $v^*(x)$ .

**Co-Authors:** Rossana Capuani, Khai T. Nguyen

**Author:** Oliver Tough

**Institution:** Duke

**Title:** The Fleming-Viot Particle System with McKean-Vlasov Dynamics

**Abstract:** The long-term behaviour of Markovian processes with an absorbing boundary has been studied since the work of Yaglom on sub-critical Galton-Watson processes. To simulate the limits we obtain in this context, Fleming-Viot particle systems were introduced by Burdzy, Holyst and March. We extend results previously established in the Markovian case to the case of McKean-Vlasov dynamics, establishing that when the appropriate long-time limits exist they are given by the scaling limits of the corresponding Fleming-Viot particle system with McKean-Vlasov dynamics. This is joint work with James Nolen.

**Co-Author:** James Nolen

**Author:** Ben Hollering

**Institution:** NCSU

**Title:** Identifiability in Phylogenetics using Algebraic Matroids

**Abstract:** A statistical model is identifiable if the parameters that produce a probability distribution in the model can be recovered from the distribution. In phylogenetics, the identifiability of the tree parameter is of particular interest. In this talk we discuss a new computational strategy for proving the identifiability of discrete parameters in algebraic statistical models that uses algebraic matroids naturally associated to the models. Specifically, we will discuss an application of this method to 2-tree mixtures for the CFN and K3P models.

**Co-Author:** Seth Sullivant

**Author:** Jane Coons

**Institution:** NCSU

**Title:** Maximum Likelihood Estimation in Independence Models with Structural Zeros

**Abstract:** A two-way independence model with structural zeros is obtained from a two-way independence model by forcing the probability of some outcomes to be zero. We use algebra and combinatorics to approach the problem of maximum likelihood estimation in these models. In cases where the structural zeros of the model avoid certain patterns, we give an explicit formula for the maximum likelihood estimate as a rational function of the data.

**Co-Author:** Seth Sullivant